

Machine Learning for Healthcare

6.871, HST.956

Lecture 10: Causal Inference Part 1

David Sontag



Recall from lecture 3...

Patient/Provider Goals of Clinical Data Science

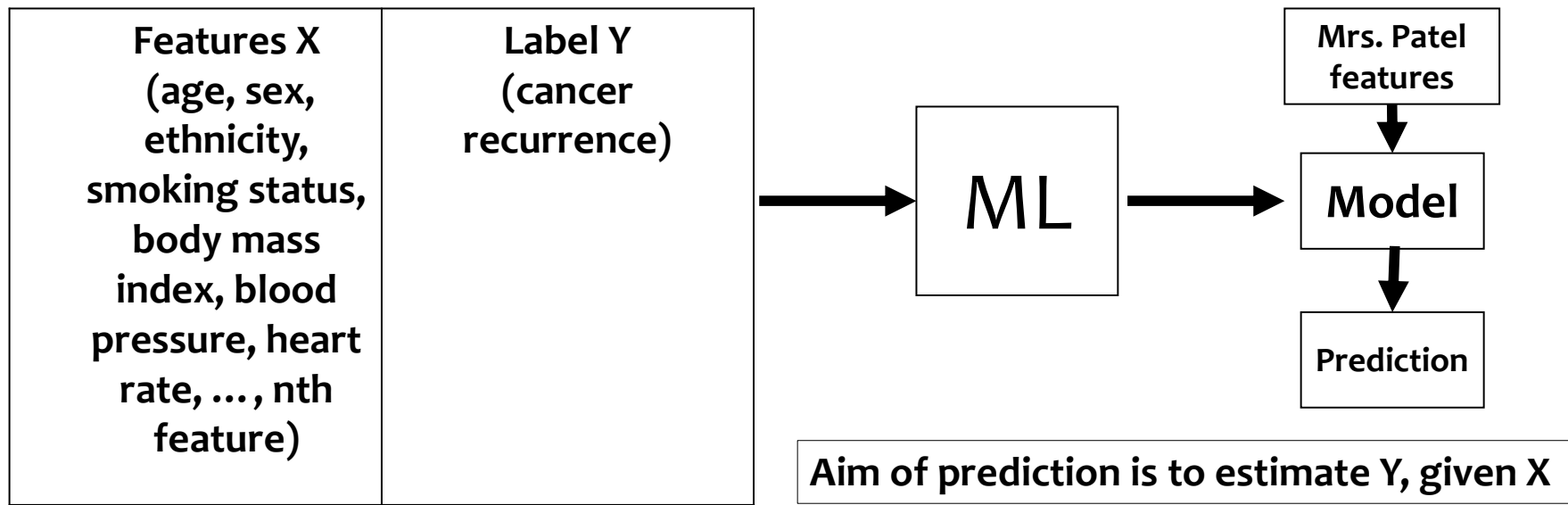
- Mrs. Patel is a 65 year old who was recently diagnosed with kidney cancer. She returns to your office to discuss treatment and has some questions.
 - After treatment, **what is the risk** of my cancer coming back before the Ultimate World Cruise (December 2023)?
 - **Will the risk** of my cancer coming back **change** if I get a partial nephrectomy instead of a radical nephrectomy?

How would you answer these questions using clinical data science?

Recall from lecture 3...

Will my cancer come back?

- How would you estimate the risk of cancer recurrence?

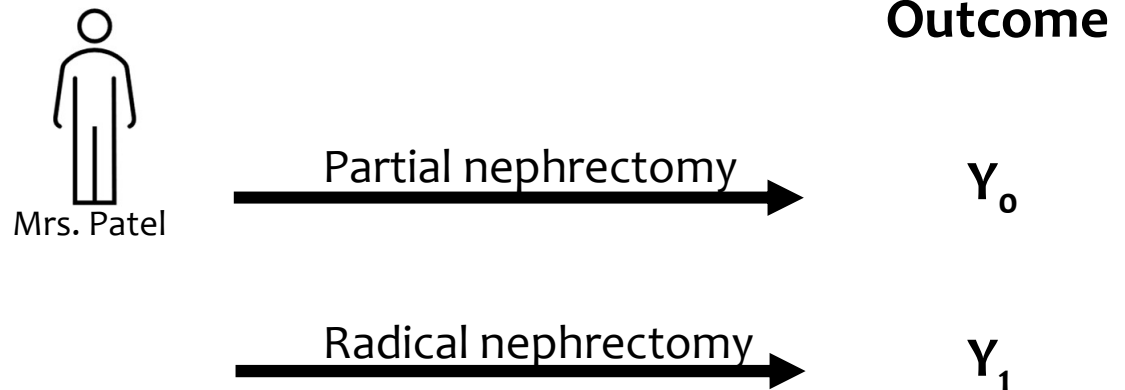


Recall from lecture 3...

Change the risk of my cancer coming back?

- You hypothesize that type of surgery (partial vs. radical) will change her risk of cancer recurrence.

- Ground truth

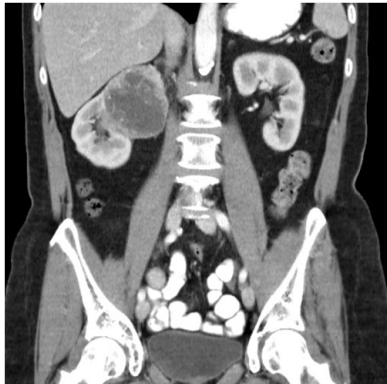


- Reality: We cannot know the ground truth

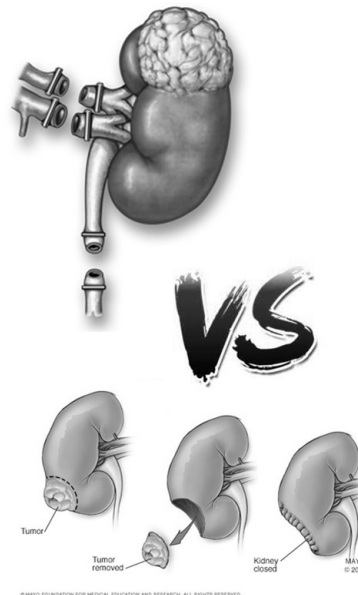
Recall from lecture 3...

RCT: Radical vs. Partial Nephrectomy

- EORTC 30904



Population: 541
patients with tumors
<5cm suspicious for
kidney cancer



Randomized to
RN vs. PN

Results

Local recurrence

RN 1/273 = 0.37%

PN 6/278 = 2.16%

Van Poppel, Hendrik, et al. "A prospective, randomised EORTC intergroup phase 3 study comparing the oncologic outcome of elective nephron-sparing surgery and radical nephrectomy for low-stage renal cell carcinoma." *European urology* 59.4 (2011): 543-552.

<https://www.fairbanksurology.com/robotic-radical-nephrectomy>
<https://www.mayoclinic.org/tests-procedures/nephrectomy/multimedia/img-20332175>

Conclusion from randomized control trial:

On average, radical nephrectomy has a *lower* rate of local recurrence than partial

Recall from lecture 3...

Clinical Research Study Designs

Descriptive

- Case report
- Case series
- Survey

Analytic

Observational

- Cohort studies
 - Cross sectional
 - Case-control

Experimental

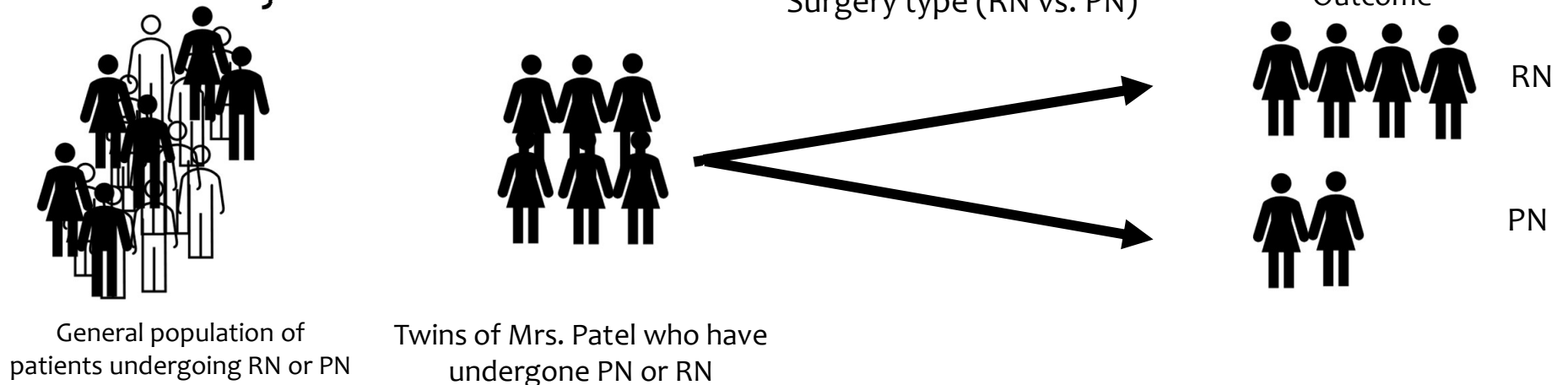
- Randomized controlled trials

Recall from lecture 3...

Change the risk of my cancer coming back?

- You hypothesize that type of surgery (partial vs. radical) will change her risk of cancer recurrence. How do you evaluate this hypothesis?

- Ideally

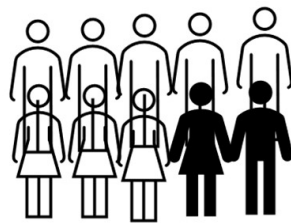
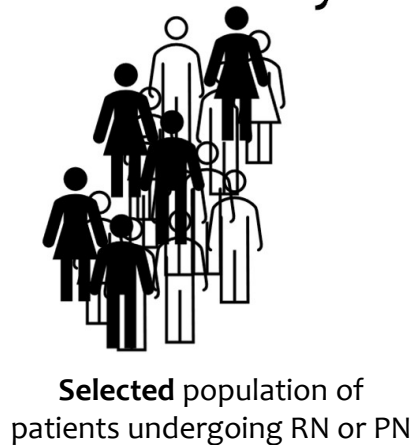


Recall from lecture 3...

Change the risk of my cancer coming back?

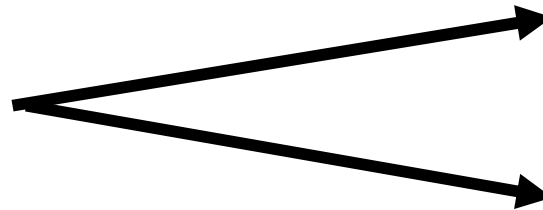
- You hypothesize that type of surgery (partial vs. radical) will change her risk of cancer recurrence. How do you evaluate this hypothesis?

- Reality

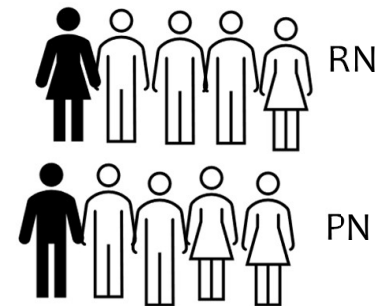


Patients “similar” to Mrs. Patel who have undergone PN or RN

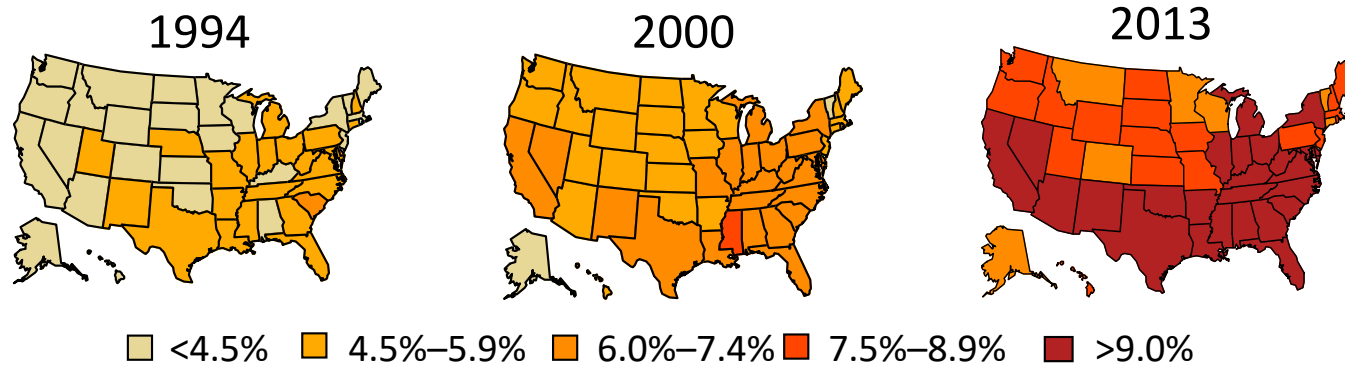
Surgery type (RN vs. PN)



Outcome

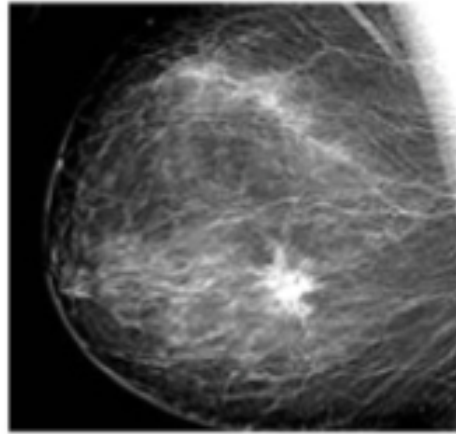


Does gastric bypass surgery prevent onset of diabetes?

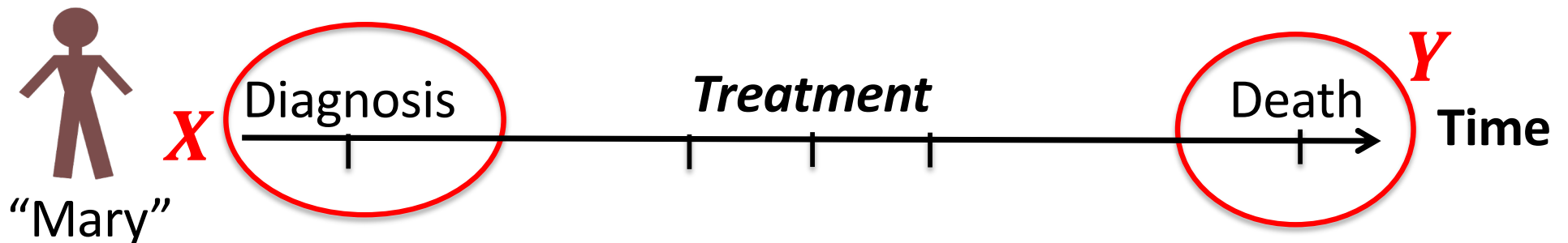


- In Lecture 4, we discussed case study of machine learning for early detection of Type 2 diabetes
- Health system doesn't want to know how to predict diabetes – they want to know how to *prevent it*
- Gastric bypass surgery is the highest negative weight (9th most predictive feature)
 - Does this mean it would be a good intervention?

What is the likelihood this patient, with breast cancer, will survive 5 years?

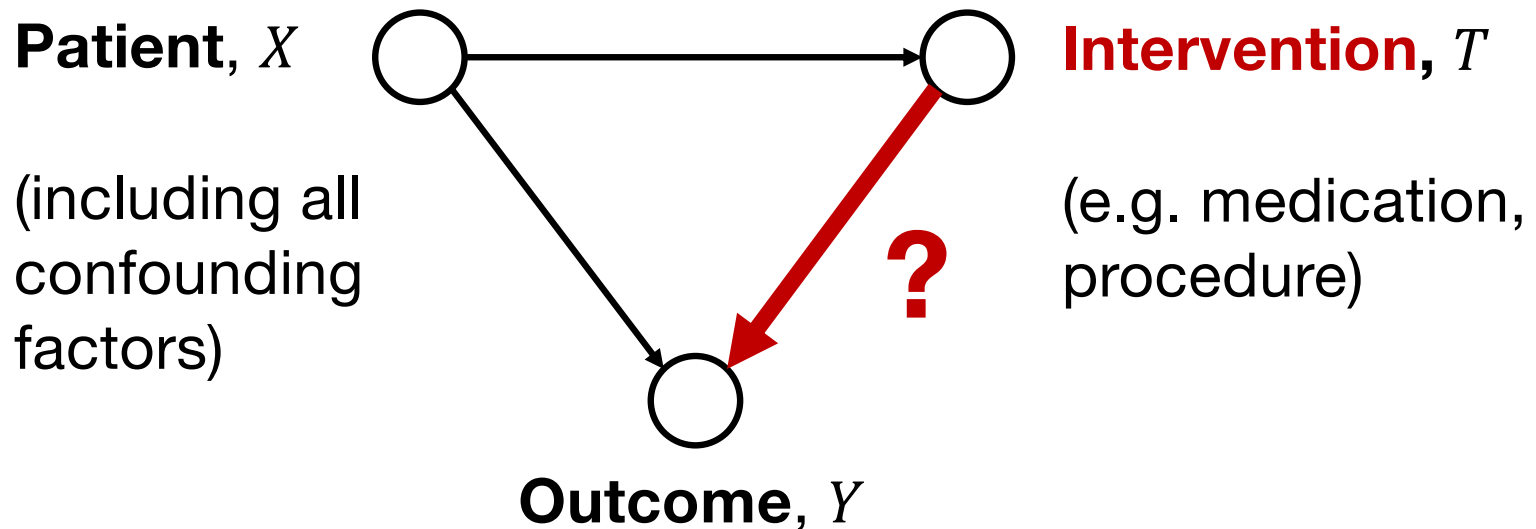


- Such predictive models widely used to stage patients. Should we initiate treatment? How aggressive?
- What could go wrong if we trained to predict survival, and then used to guide patient care?



A long survival time may be because of treatment!

To properly answer, need to formulate as *causal* questions:



High dimensional

Observational data

Potential Outcomes Framework (Rubin-Neyman Causal Model)

- Each unit (individual) x_i has two potential outcomes*:
 - $Y_0(x_i)$ is the potential outcome had the unit not been treated:
“**control outcome**”
 - $Y_1(x_i)$ is the potential outcome had the unit been treated:
“**treated outcome**”
- Conditional average treatment effect for unit i :
$$CATE(x_i) = \mathbb{E}_{Y_1 \sim p(Y_1|x_i)} [Y_1|x_i] - \mathbb{E}_{Y_0 \sim p(Y_0|x_i)} [Y_0|x_i]$$
- Average Treatment Effect:
$$ATE := \mathbb{E}[Y_1 - Y_0] = \mathbb{E}_{x \sim p(x)} [CATE(x)]$$

* other commonly used notations for Y_0 are $Y(0)$ and $Y \mid \text{do}(T)=0$

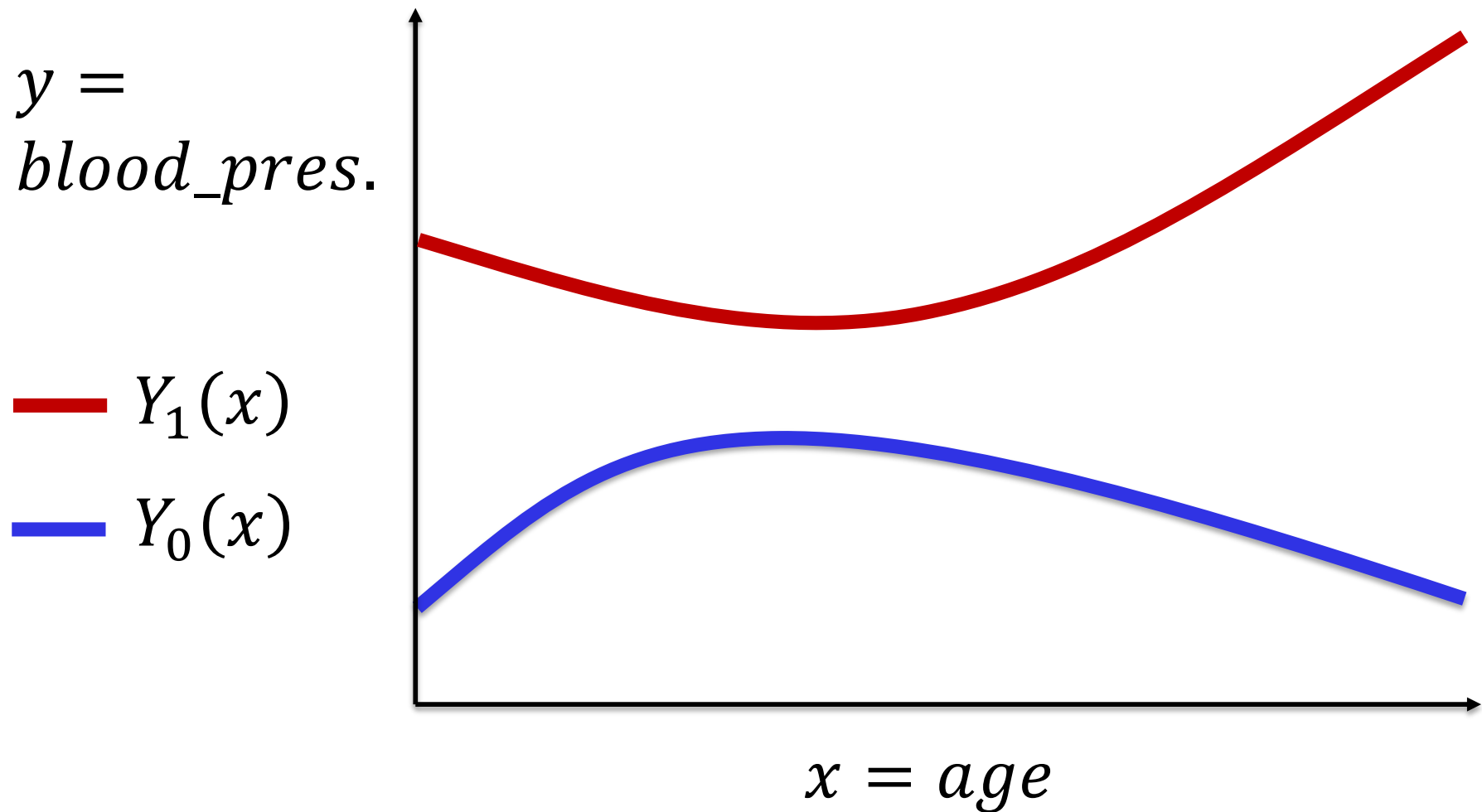
Potential Outcomes Framework (Rubin-Neyman Causal Model)

- Each unit (individual) x_i has two potential outcomes:
 - $Y_0(x_i)$ is the potential outcome had the unit not been treated:
“**control outcome**”
 - $Y_1(x_i)$ is the potential outcome had the unit been treated:
“**treated outcome**”
- Observed factual outcome:
$$y_i = t_i Y_1(x_i) + (1 - t_i) Y_0(x_i)$$
- Unobserved counterfactual outcome:
$$y_i^{CF} = (1 - t_i) Y_1(x_i) + t_i Y_0(x_i)$$

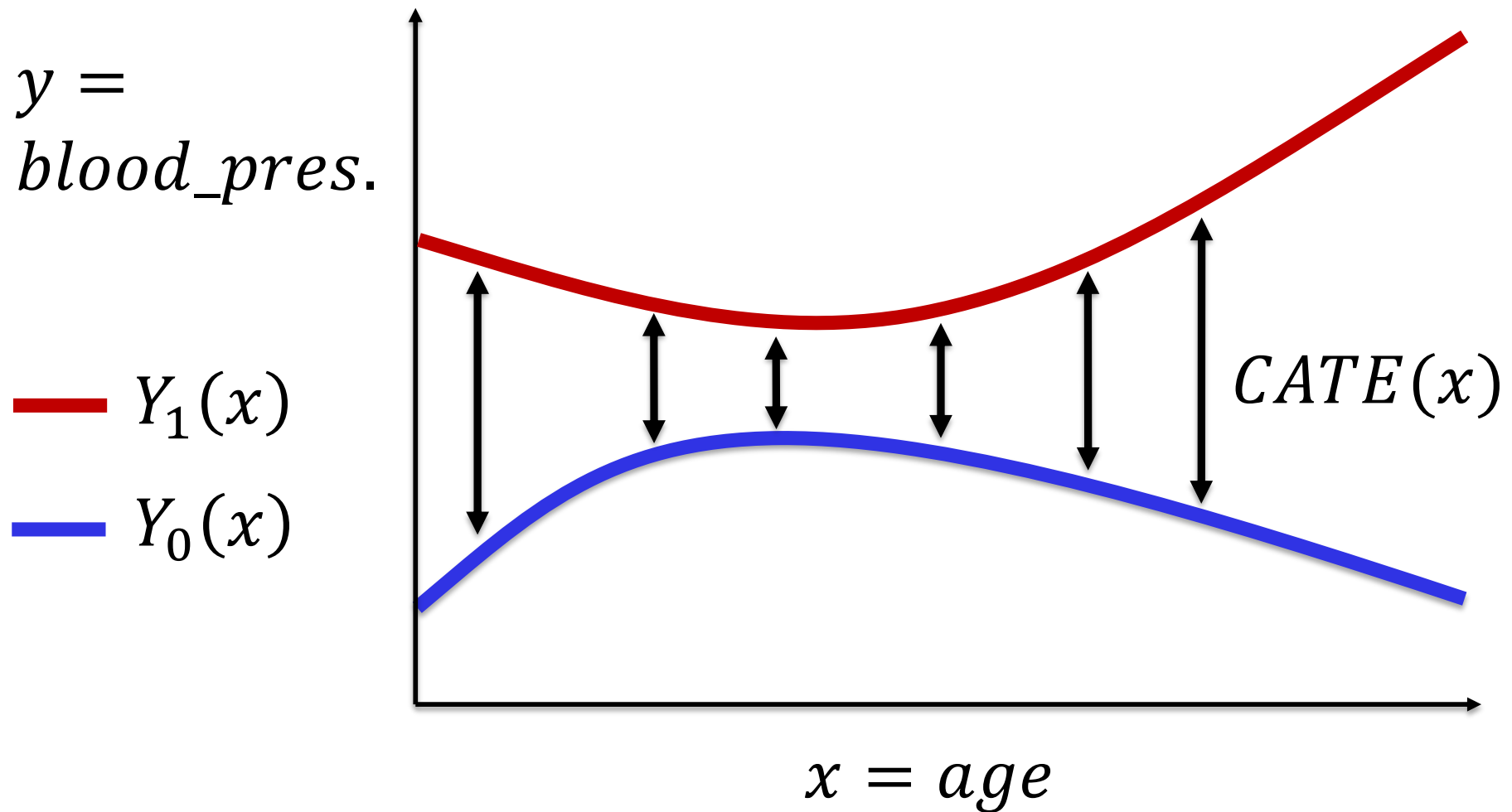
“The fundamental problem of
causal inference”

We only ever observe one of the
two outcomes

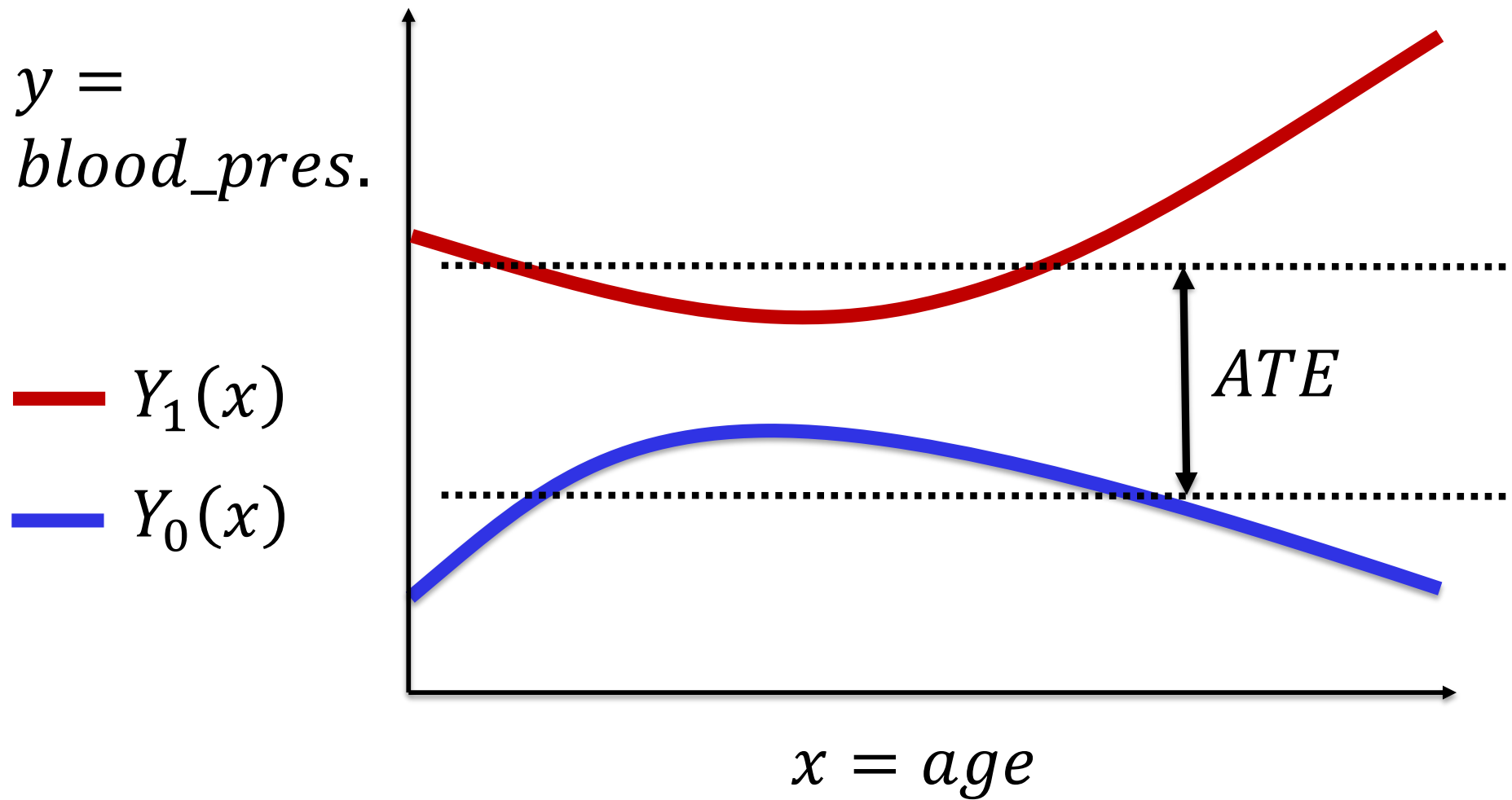
Example – Blood pressure and age



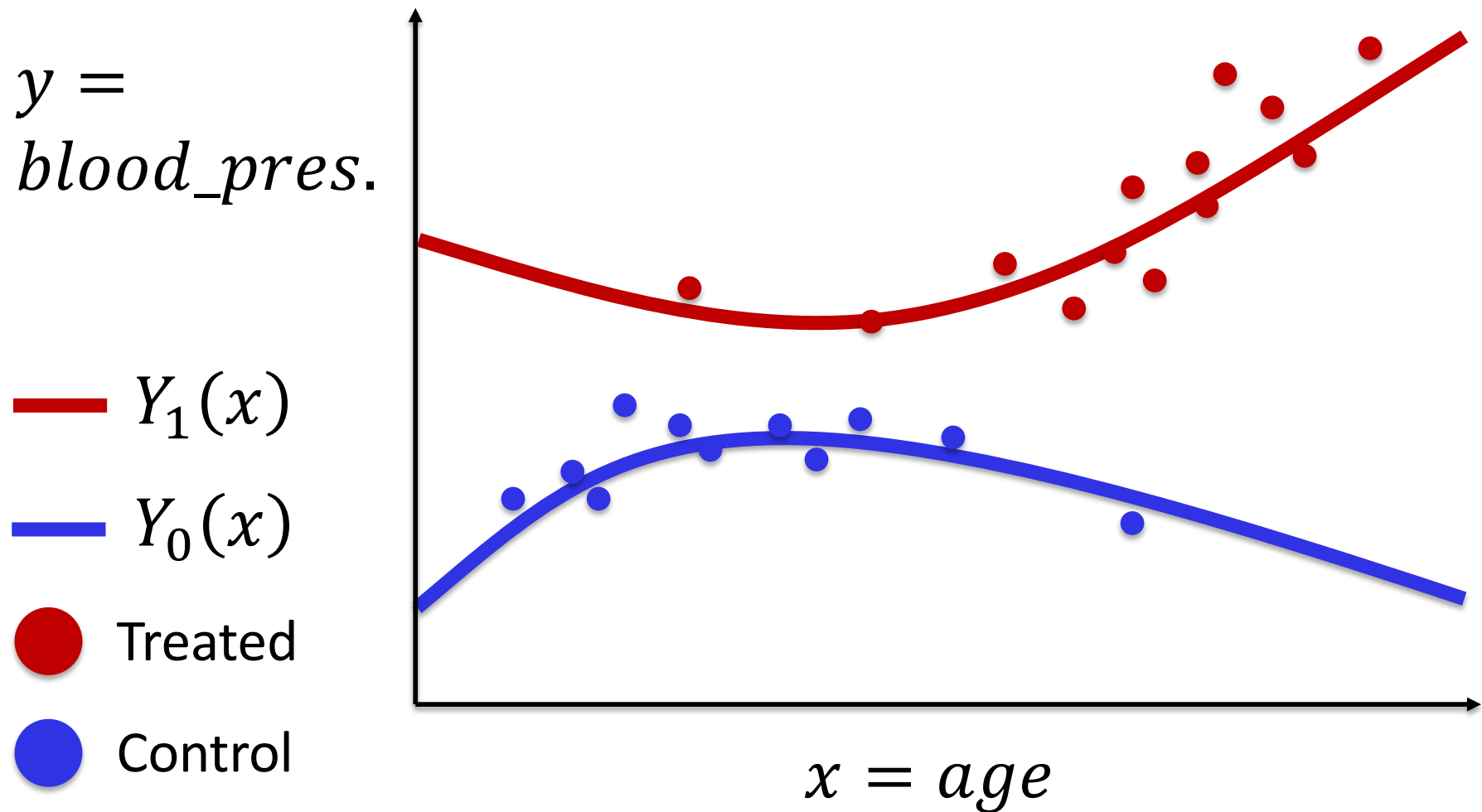
Blood pressure and age



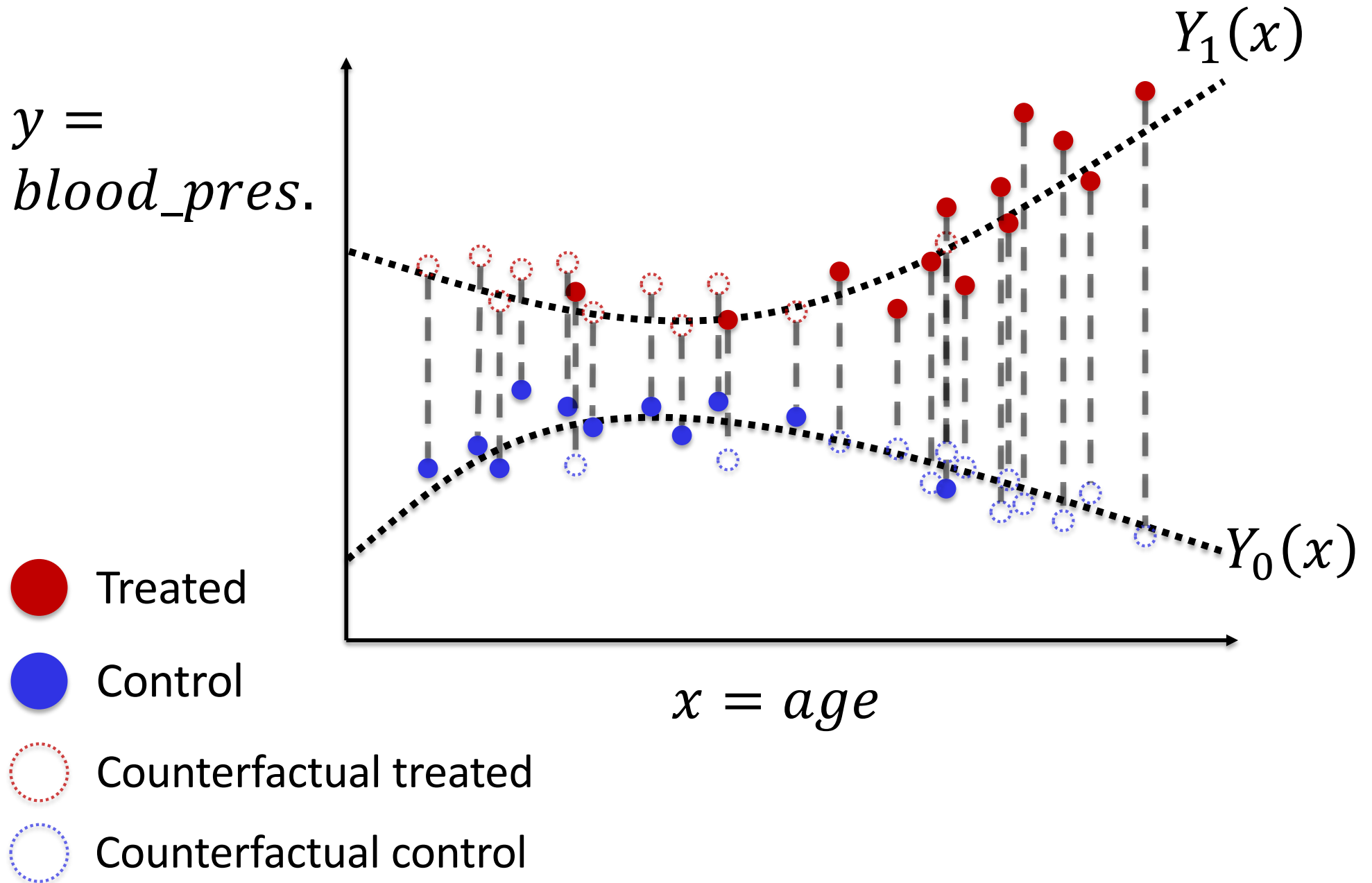
Blood pressure and age



Blood pressure and age



Blood pressure and age



(age, gender, exercise, treatment)			Observed sugar levels
(45, F, 0, A)			6
(45, F, 1, B)			6.5
(55, M, 0, A)			7
(55, M, 1, B)			8
(65, F, 0, B)			8
(65, F, 1, A)			7.5
(75, M, 0, B)			9
(75, M, 1, A)			8

(Example from Uri Shalit)

(age, gender, exercise)			Observed sugar levels
(45, F, 0)			6
(45, F, 1)			6.5
(55, M, 0)			7
(55, M, 1)			8
(65, F, 0)			8
(65, F, 1)			7.5
(75, M, 0)			9
(75, M, 1)			8

(Example from Uri Shalit)

(age, gender, exercise)	Y_0 : Sugar levels <i>had they received medication A</i>	Y_1 : Sugar levels <i>had they received medication B</i>	Observed sugar levels
(45, F, 0)	6	5.5	6
(45, F, 1)	7	6.5	6.5
(55, M, 0)	7	6	7
(55, M, 1)	9	8	8
(65, F, 0)	8.5	8	8
(65, F, 1)	7.5	7	7.5
(75, M, 0)	10	9	9
(75, M, 1)	8	7	8

(Example from Uri Shalit)

(age,gender, exercise)	Sugar levels <i>had they received medication A</i>	Sugar levels <i>had they received medication B</i>	Observed sugar levels
(45, F, 0)	6	5.5	6
(45, F, 1)	7	6.5	6.5
(55, M, 0)	7	6	7
(55, M, 1)	9	8	8
(65, F, 0)	8.5	8	8
(65, F, 1)	7.5	7	7.5
(75, M, 0)	10	9	9
(75, M, 1)	8	7	8

mean(sugar | medication B) –
mean(sugar | medication A) =
?

mean(sugar | *had they received B*) –
mean(sugar | *had they received A*) =
?

(Example from Uri Shalit)

Most important assumption – no unmeasured confounders

Y_0, Y_1 : potential outcomes for control and treated

x : unit covariates (features)

T : treatment assignment

We assume:

$$(Y_0, Y_1) \perp\!\!\!\perp T \mid x$$

The potential outcomes are independent of treatment assignment, conditioned on covariates x

Most important assumption – no unmeasured confounders

Y_0, Y_1 : potential outcomes for control and treated

x : unit covariates (features)

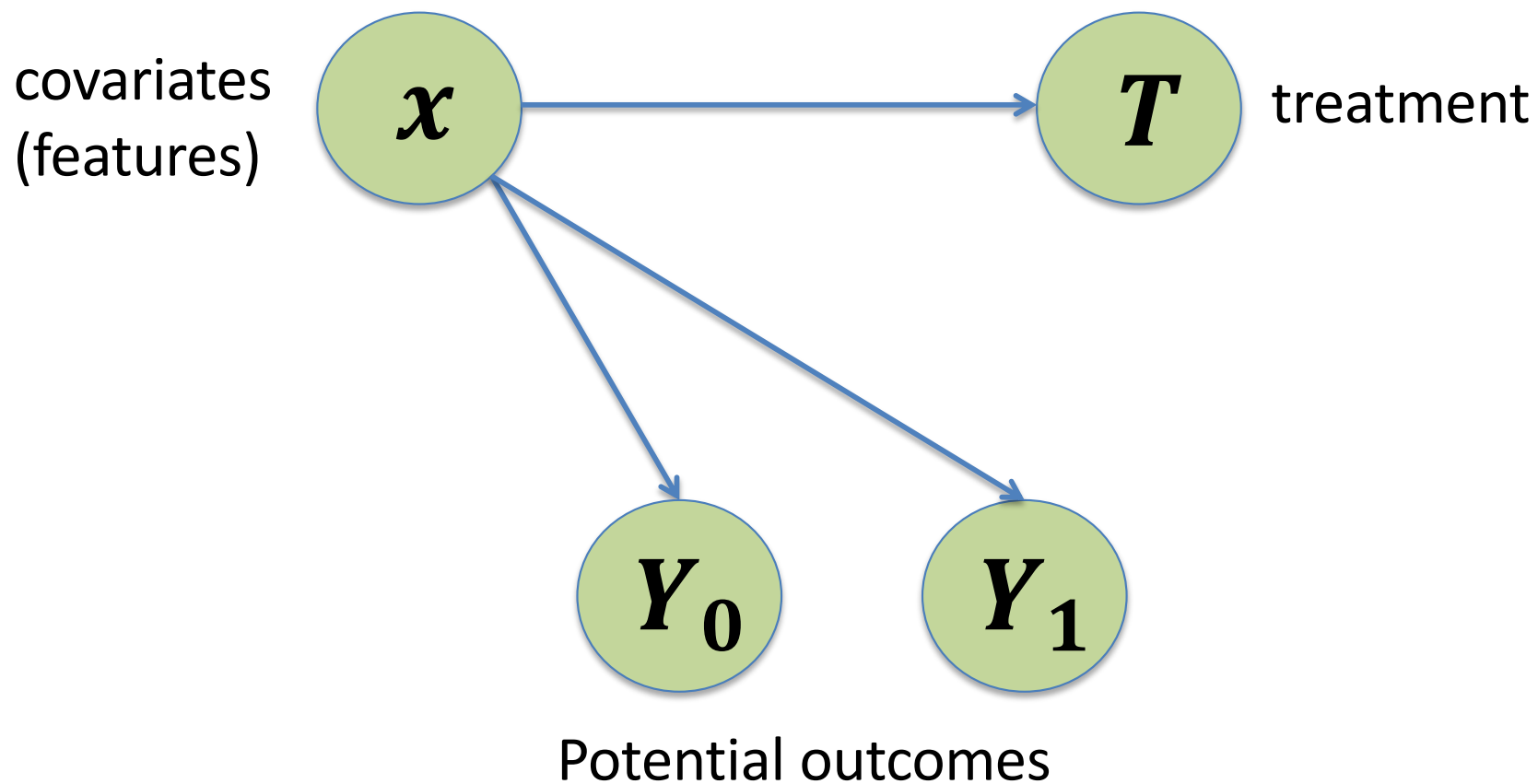
T : treatment assignment

We assume:

$$(Y_0, Y_1) \perp\!\!\!\perp T \mid x$$

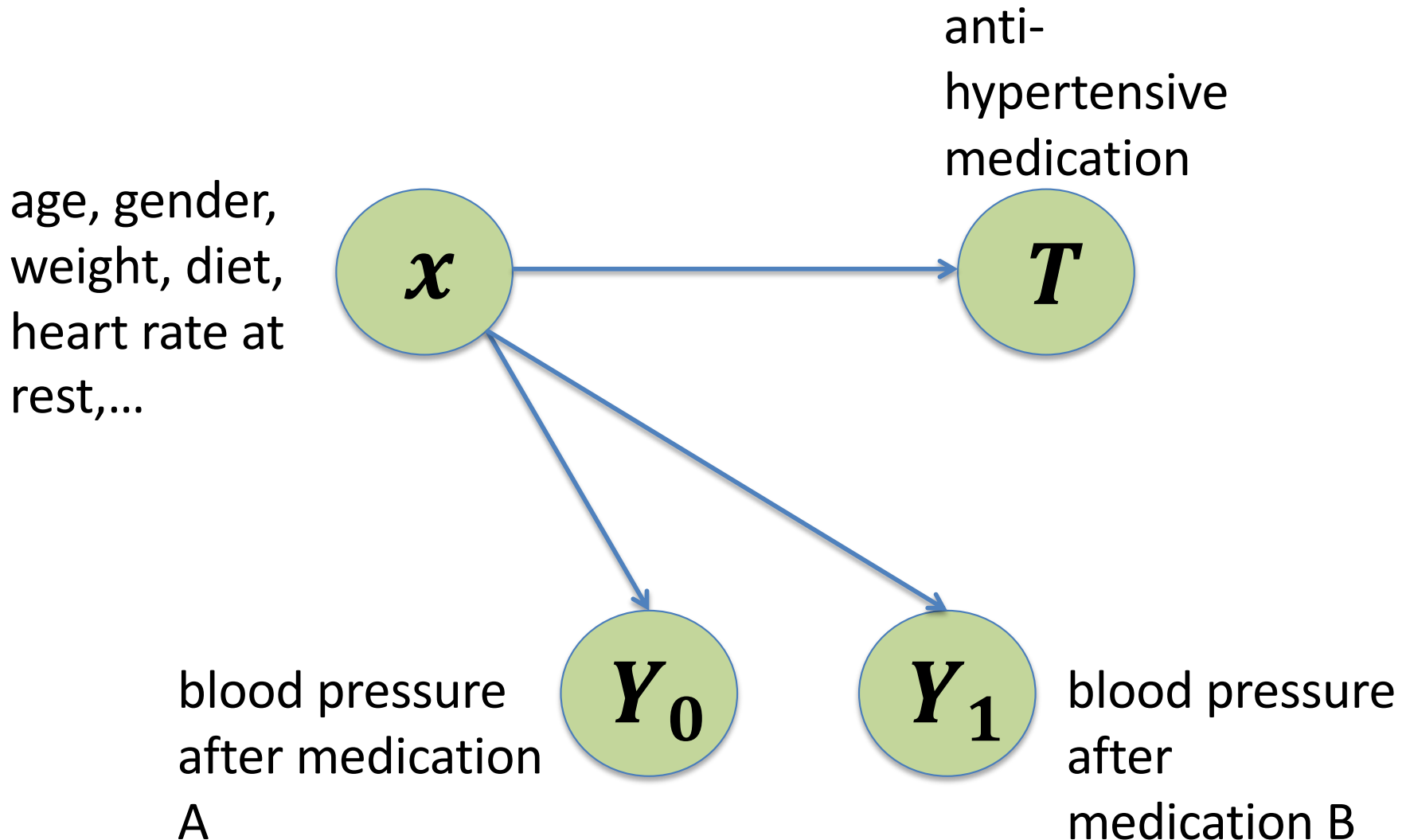
Ignorability

Ignorability



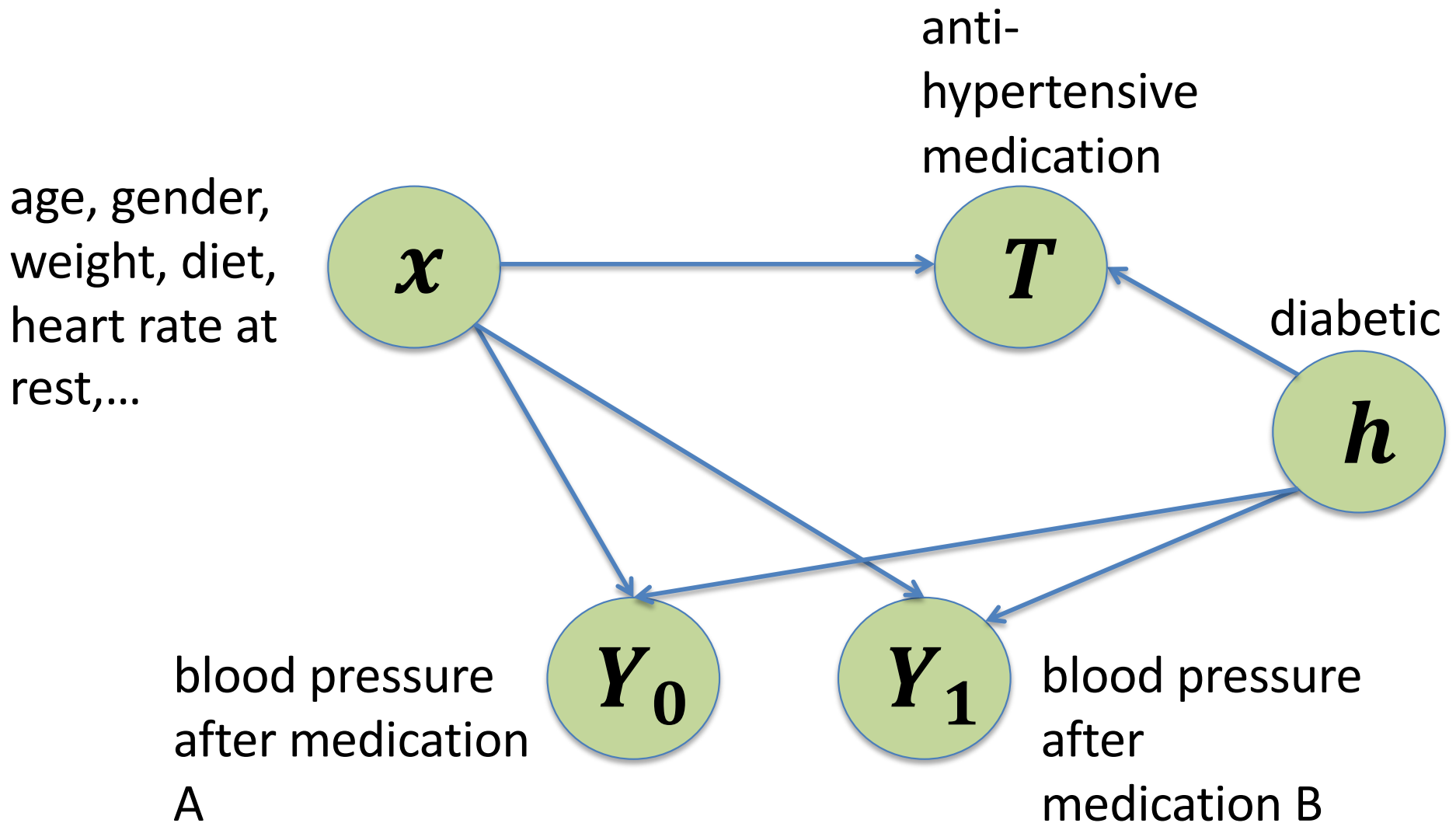
$$(Y_0, Y_1) \perp\!\!\!\perp T \mid x$$

Ignorability

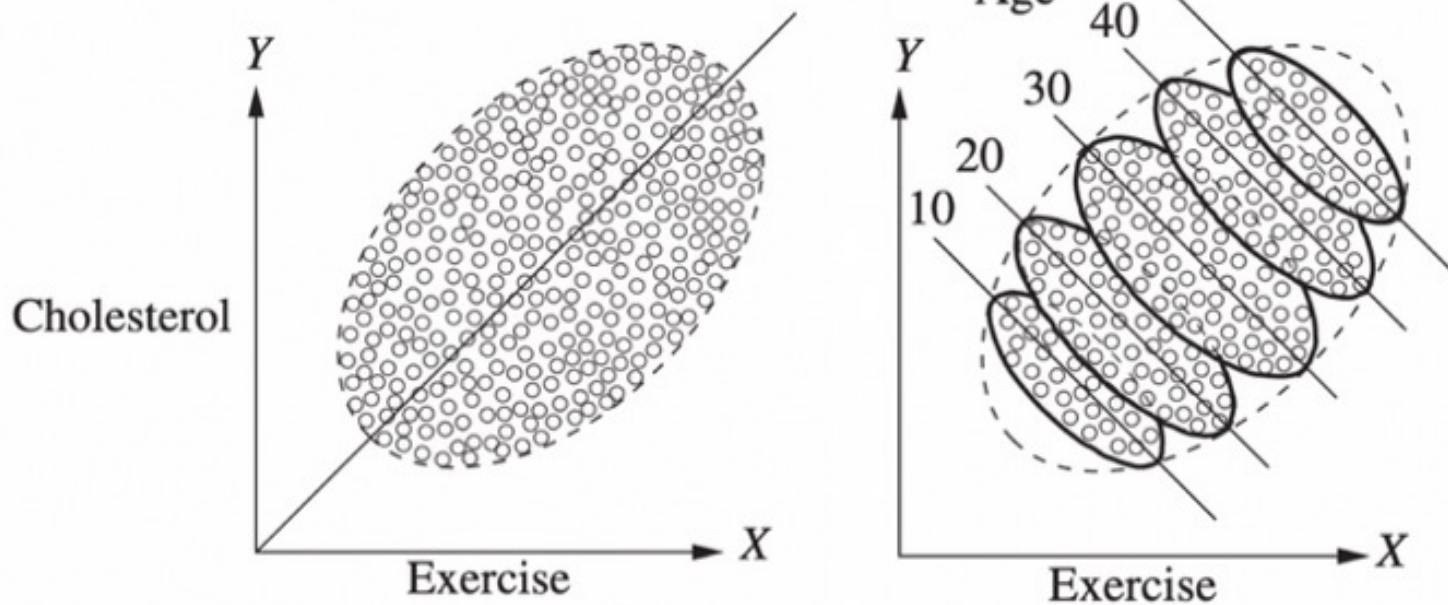


$$(Y_0, Y_1) \perp\!\!\!\perp T \mid X$$

No Ignorability



$$(Y_0, Y_1) \not\perp T \mid x$$



Weekly exercise effect on cholesterol?

(Pearl et al. 2016)

Outline for lecture

- How to recognize a causal inference problem
- Potential outcomes framework
 - Average treatment effect (ATE)
 - Conditional average treatment effect (CATE)
- **Covariate adjustment: A method for estimating ATE and CATE**
- Theory – when/why does this work?

Many methods!

Covariate adjustment

Propensity score re-weighting

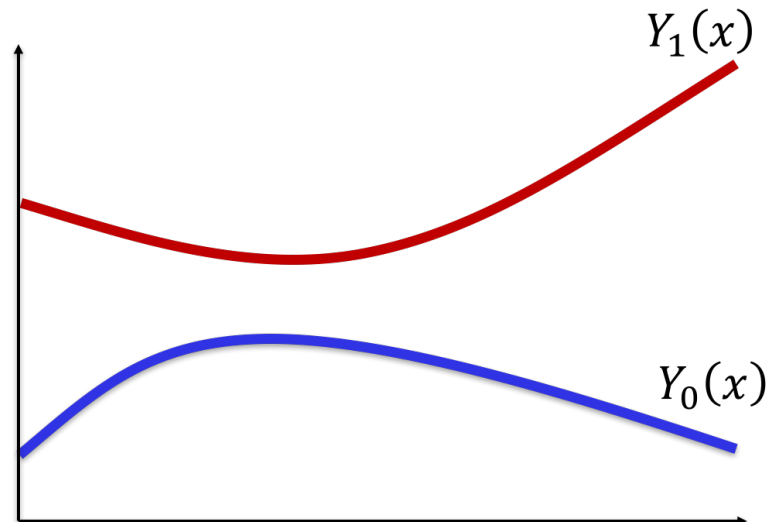
Doubly robust estimators

Matching

...

Covariate adjustment

- Explicitly model the relationship between treatment, confounders, and outcome
- Also called “Response Surface Modeling”
- Used for both CATE and ATE
- A regression problem



Covariate adjustment

- Explicitly model the relationship between treatment, confounders, and outcome
- Under ignorability, the expected causal effect of T on Y :

$$\mathbb{E}_{x \sim p(x)} \left[\mathbb{E}[Y|T = 1, x] - \mathbb{E}[Y|T = 0, x] \right]$$

- Fit a model $f(x, t) \approx \mathbb{E}[Y|T = t, x]$

$$\widehat{ATE} = \frac{1}{n} \sum_{i=1}^n \left(f(x^i, 1) - f(x^i, 0) \right)$$

$$\widehat{CATE}(x^i) = f(x^i, 1) - f(x^i, 0)$$

Covariates
(Features)

x_1

x_2

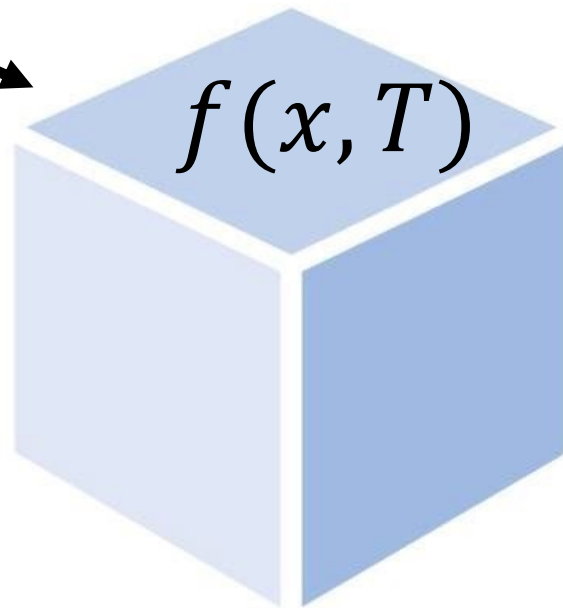
⋮

x_d

Treatment
(0/1)

T

Regression
model



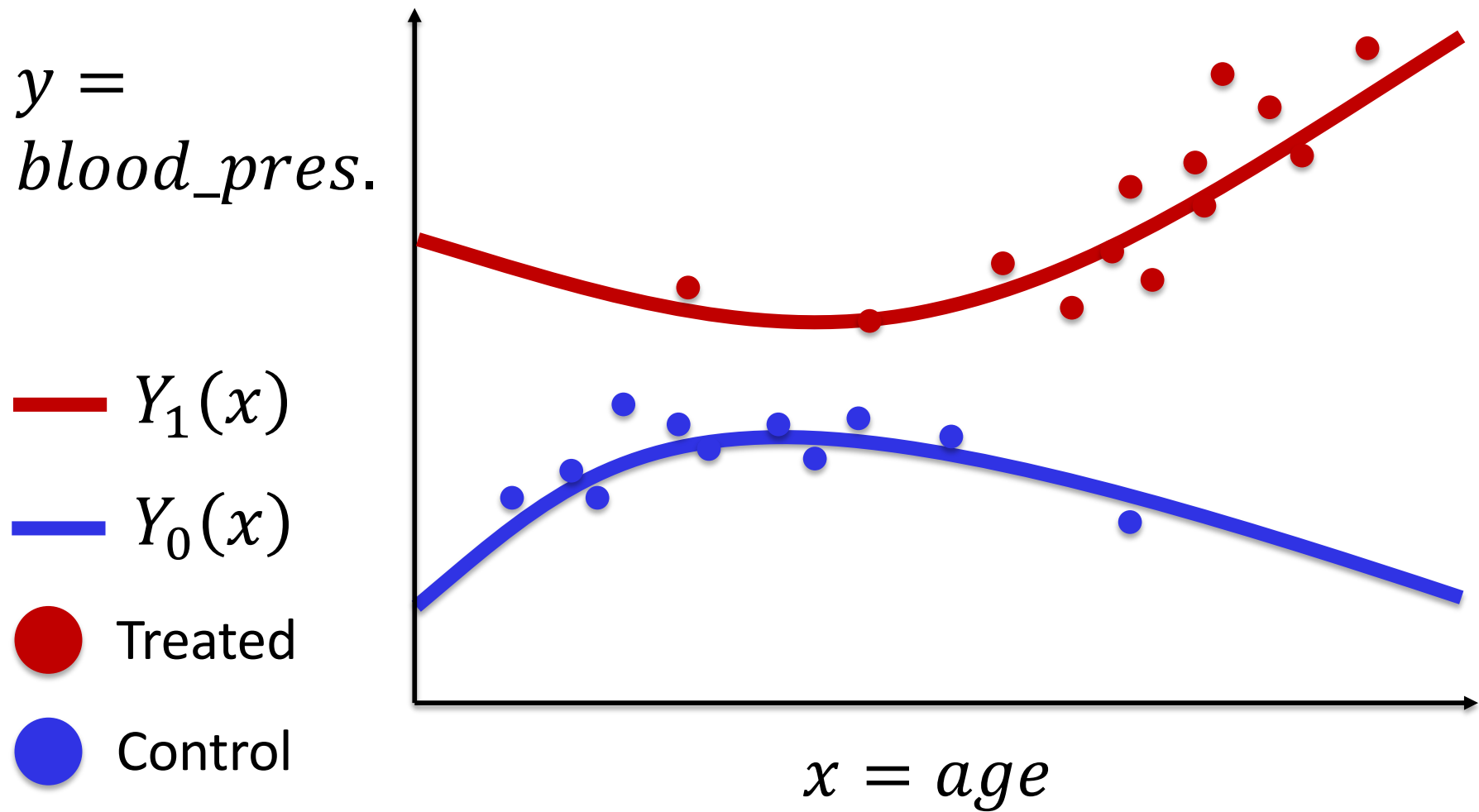
Outcome

y

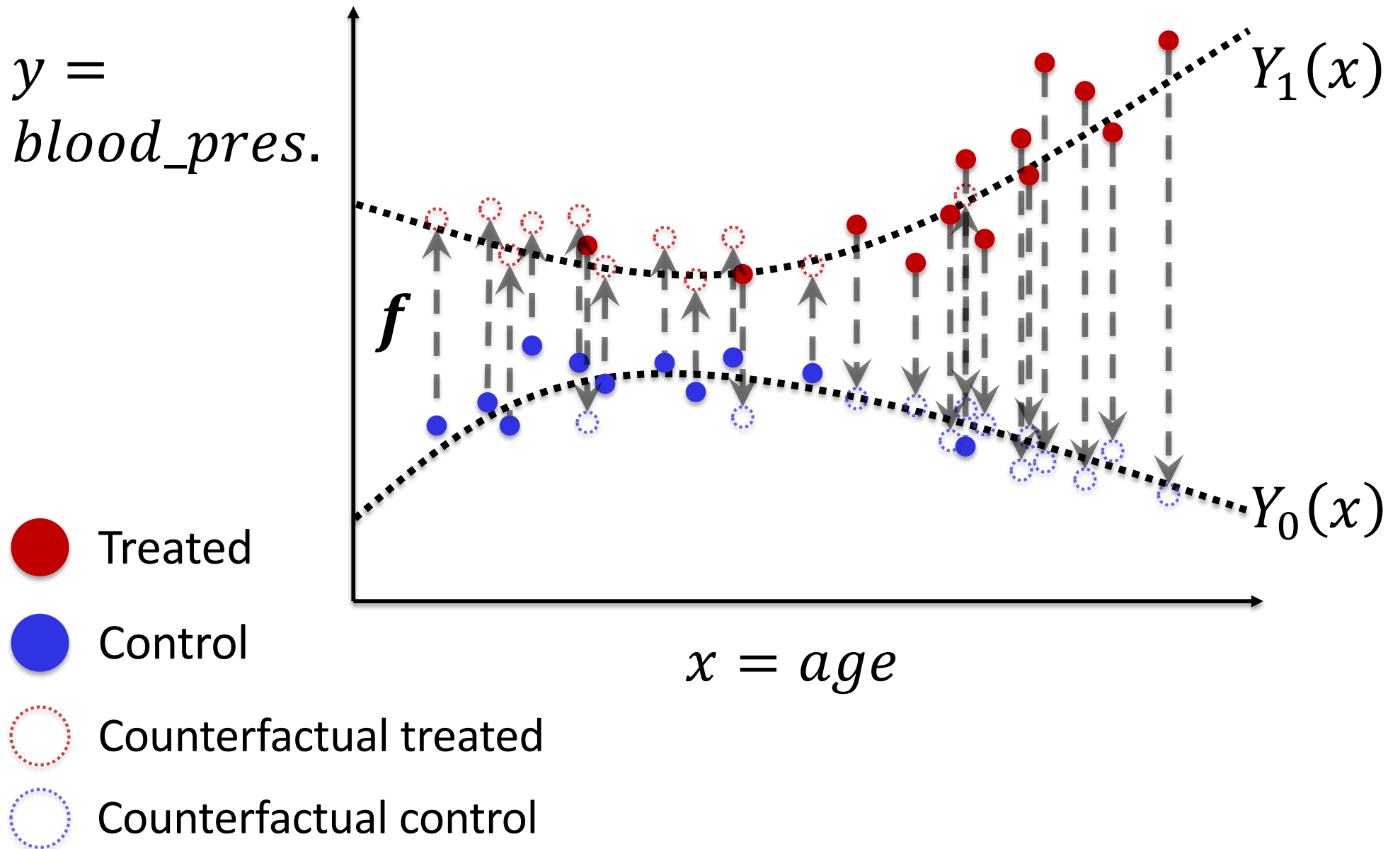
Fit a model $f(x, t) \approx \mathbb{E}[Y|X, T]$

$$\widehat{ATE} = \frac{1}{n} \sum_{i=1}^n f(x^i, 1) - f(x^i, 0)$$

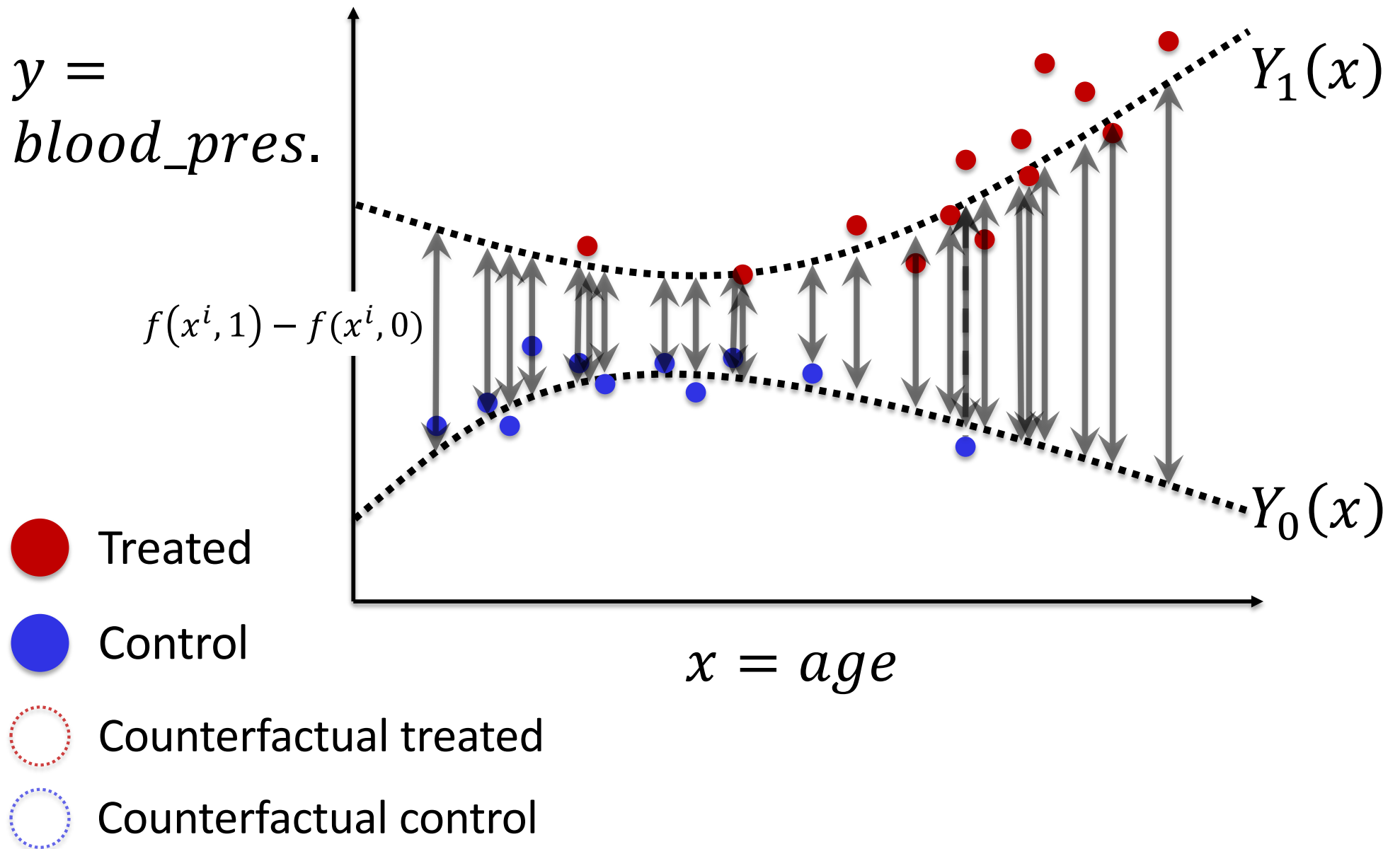
Recall this example...



Covariate adjustment (intuition): imputing the counterfactual



Covariate adjustment (reality): estimating difference in means



Outline for lecture

- How to recognize a causal inference problem
- Potential outcomes framework
 - Average treatment effect (ATE)
 - Conditional average treatment effect (CATE)
- Covariate adjustment: A method for estimating ATE and CATE
- **Theory – when/why does this work?**

Average Treatment Effect – the adjustment formula

- Assuming ignorability, we will derive the *adjustment formula* (Hernán & Robins 2010, Pearl 2009)
- Also called *(parametric) G-formula*

$$ATE = \mathbb{E} [Y_1 - Y_0] =$$

$$\mathbb{E}_{x \sim p(x)} [\mathbb{E} [Y_1 | x, T = 1] - \mathbb{E} [Y_0 | x, T = 0]]$$

Average Treatment Effect

The expected causal effect of T on Y :

$$ATE := \mathbb{E} [Y_1 - Y_0]$$

Average Treatment Effect

The expected causal effect of T on Y :

$$ATE := \mathbb{E} [Y_1 - Y_0]$$

$$\mathbb{E} [Y_1] =$$

law of total
expectation

$$\mathbb{E}_{x \sim p(x)} \left[\mathbb{E}_{Y_1 \sim p(Y_1|x)} [Y_1 | x] \right] =$$

Average Treatment Effect

The expected causal effect of T on Y :

$$ATE := \mathbb{E} [Y_1 - Y_0]$$

$$\mathbb{E} [Y_1] =$$

$$\mathbb{E}_{x \sim p(x)} \left[\mathbb{E}_{Y_1 \sim p(Y_1 | x)} [Y_1 | x] \right] = \text{ignorability} \\ (Y_0, Y_1) \perp\!\!\!\perp T | x$$

$$\mathbb{E}_{x \sim p(x)} \left[\mathbb{E}_{Y_1 \sim p(Y_1 | x, T=1)} [Y_1 | x, T = 1] \right] =$$

Average Treatment Effect

The expected causal effect of T on Y :

$$ATE := \mathbb{E} [Y_1 - Y_0]$$

$$\mathbb{E} [Y_1] =$$

$$\mathbb{E}_{x \sim p(x)} \left[\mathbb{E}_{Y_1 \sim p(Y_1 | x)} [Y_1 | x] \right] =$$

$$\mathbb{E}_{x \sim p(x)} \left[\mathbb{E}_{Y_1 \sim p(Y_1 | x, T=1)} [Y_1 | x, T = 1] \right] =$$

$$\mathbb{E}_{x \sim p(x)} \left[\mathbb{E} [Y_1 | x, T = 1] \right] \quad \text{shorter notation}$$

Average Treatment Effect

The expected causal effect of T on Y :

$$ATE := \mathbb{E} [Y_1 - Y_0]$$

$$\mathbb{E} [Y_0] =$$

$$\mathbb{E}_{x \sim p(x)} \left[\mathbb{E}_{Y_0 \sim p(Y_0|x)} [Y_0|x] \right] =$$

$$\mathbb{E}_{x \sim p(x)} \left[\mathbb{E}_{Y_0 \sim p(Y_0|x, T=0)} [Y_0|x, T=0] \right] =$$

$$\mathbb{E}_{x \sim p(x)} \left[\mathbb{E} [Y_0|x, T = 0] \right]$$

The adjustment formula

Under the assumption of ignorability, we have that:

$$ATE = \mathbb{E} [Y_1 - Y_0] =$$

$$\mathbb{E}_{x \sim p(x)} [\mathbb{E} [Y_1 | x, T = 1] - \mathbb{E} [Y_0 | x, T = 0]]$$

$\mathbb{E} [Y_1 | x, T = 1]$
 $\mathbb{E} [Y_0 | x, T = 0]$ } Quantities we
can estimate
from data

The adjustment formula

Under the assumption of ignorability, we have that:

$$ATE = \mathbb{E} [Y_1 - Y_0] = \mathbb{E}_{x \sim p(x)} [\mathbb{E} [Y_1 | x, T = 1] - \mathbb{E} [Y_0 | x, T = 0]]$$

$$\mathbb{E} [Y_0 | x, T = 1]$$

$$\mathbb{E} [Y_1 | x, T = 0]$$

$$\mathbb{E} [Y_0 | x]$$

$$\mathbb{E} [Y_1 | x]$$

Quantities we
cannot directly
estimate from data

Covariates
(Features)

X_1

X_2

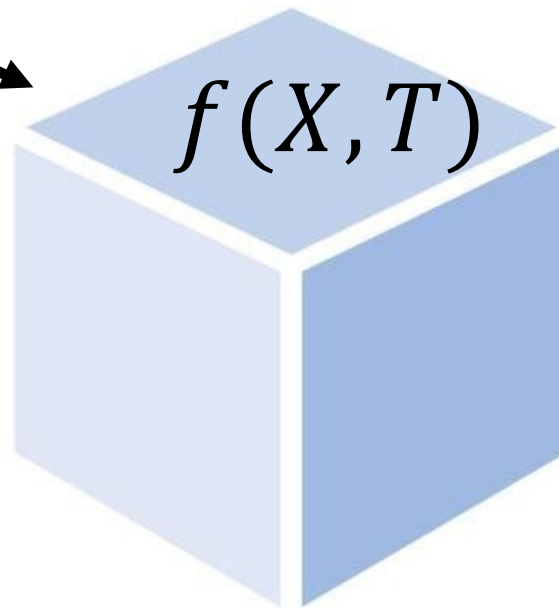
⋮

X_d

Treatment
(0/1)

T

Regression
model



Outcome

y

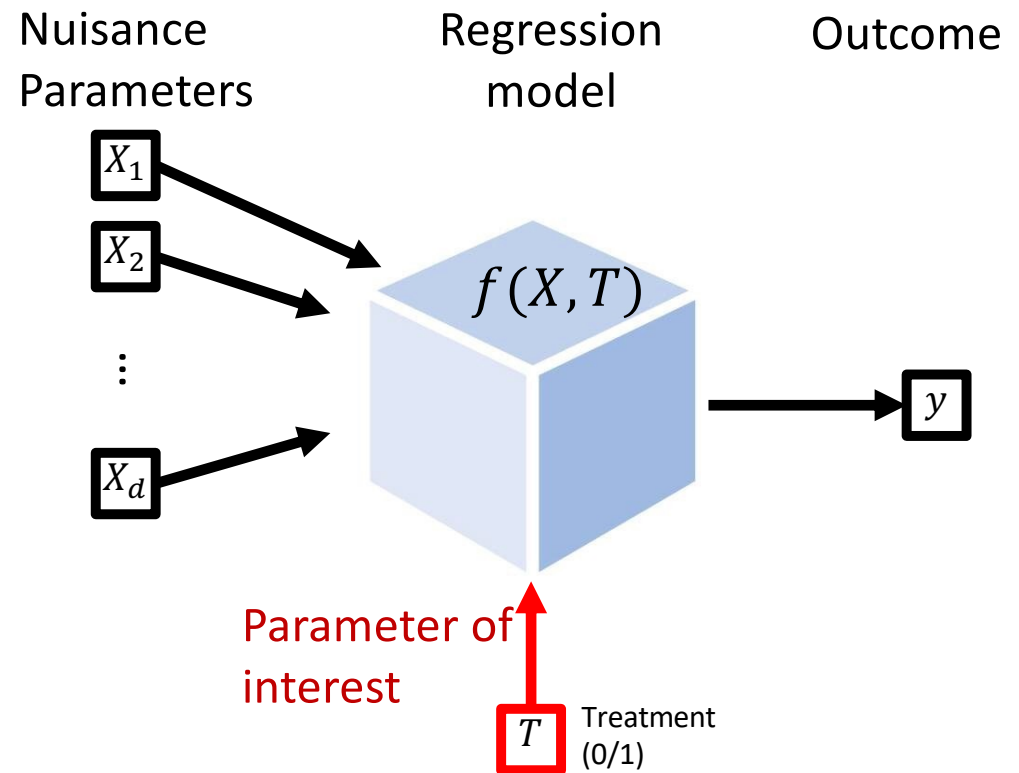
Fit a model $f(x, t) \approx \mathbb{E}[Y|X, T]$

$$\widehat{ATE} = \frac{1}{n} \sum_{i=1}^n f(x^i, 1) - f(x^i, 0)$$

Straightforward
application of
machine learning,
right?

Covariate adjustment relies on being able to extrapolate correctly

- Correctly estimating the (C)ATE depends on being able to tell the difference between $T=1$ and $T=0$
- Either need to make strong parametric assumptions about the form of $\mathbb{E}[Y|X, T]$, or
- Make no assumptions about form of $\mathbb{E}[Y|X, T]$ (use black-box ML method for f); instead, make assumptions about $p(t | x)$



$$\widehat{ATE} = \frac{1}{n} \sum_{i=1}^n f(x^i, 1) - f(x^i, 0)$$

Summary

- One approach to use machine learning for causal inference
 - Predict outcome given features and treatment, then use resulting model to impute counterfactuals (*covariate adjustment*)
- Consistency of estimates depend on:
 - Causal graph being correct (i.e., no **unobserved confounding**)
 - Identifiability of causal effect; more on this in Thursday's lecture

References

- Recent work from ML community:
<https://sites.google.com/view/nips2018causallearning/> and
http://tripods.cis.cornell.edu/neurips19_causalml/
- Recent book on causal inference by Miguel Hernan and Jamie Robins:
<https://www.hsph.harvard.edu/miguel-hernan/causal-inference-book/>
Recent book on causal inference by Jonas Peters, Dominik Janzing and Bernhard Schölkopf:
<https://mitpress.mit.edu/books/elements-causal-inference>
(download PDF for free on left: “Open Access Title”)
- Examples of recent papers in this research field:
<https://arxiv.org/abs/1906.02120>
<https://arxiv.org/abs/1705.08821>
<https://arxiv.org/abs/1510.04342>
<https://arxiv.org/abs/1810.02894>